



Oxford Cambridge and RSA

**Monday 05 October 2020 – Afternoon**

**AS Level Further Mathematics A**

**Y531/01 Pure Core**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **4** pages.

**ADVICE**

- Read each question carefully before you start your answer.

Answer **all** the questions.

**1 In this question you must show detailed reasoning.**

Use an algebraic method to find the square roots of  $-77 - 36i$ . [6]

**2** P, Q and T are three transformations in 2-D.

P is a reflection in the  $x$ -axis. **A** is the matrix that represents P.

(a) Write down the matrix **A**. [1]

Q is a shear in which the  $y$ -axis is invariant and the point  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  is transformed to the point  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ . **B** is the matrix that represents Q.

(b) Find the matrix **B**. [2]

T is P followed by Q. **C** is the matrix that represents T.

(c) Determine the matrix **C**. [2]

$L$  is the line whose equation is  $y = x$ .

(d) Explain whether or not  $L$  is a line of invariant points under  $T$ . [2]

An object parallelogram,  $M$ , is transformed under T to an image parallelogram,  $N$ .

(e) Explain what the value of the determinant of **C** means about

- the area of  $N$  compared to the area of  $M$ ,
- the orientation of  $N$  compared to the orientation of  $M$ .

[3]

**3 In this question you must show detailed reasoning.**

The complex number  $7 - 4i$  is denoted by  $z$ .

(a) Giving your answers in the form  $a + bi$ , where  $a$  and  $b$  are rational numbers, find the following.

(i)  $3z - 4z^*$  [2]

(ii)  $(z + 1 - 3i)^2$  [2]

(iii)  $\frac{z+1}{z-1}$  [2]

(b) Express  $z$  in modulus-argument form giving the modulus exactly and the argument correct to 3 significant figures. [3]

(c) The complex number  $\omega$  is such that  $z\omega = \sqrt{585}(\cos(0.5) + i\sin(0.5))$ .

Find the following.

- $|\omega|$
- $\arg(\omega)$ , giving your answer correct to 3 significant figures [3]

**4 You are given the system of equations**

$$a^2x - 2y = 1$$

$$x + b^2y = 3$$

where  $a$  and  $b$  are real numbers.

(a) Use a matrix method to find  $x$  and  $y$  in terms of  $a$  and  $b$ . [4]

(b) Explain why the method used in part (a) works for all values of  $a$  and  $b$ . [2]

**5 In this question you must show detailed reasoning.**

The cubic equation  $5x^3 + 3x^2 - 4x + 7 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .

Find a cubic equation with integer coefficients whose roots are  $\alpha + \beta$ ,  $\beta + \gamma$  and  $\gamma + \alpha$ . [7]

6 Prove that  $n! > 2^{2n}$  for all integers  $n \geq 9$ . [5]

7 The equations of two **intersecting** lines are

$$\mathbf{r} = \begin{pmatrix} -12 \\ a \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$$

where  $a$  is a constant.

(a) Find a vector,  $\mathbf{b}$ , which is perpendicular to both lines. [2]

(b) Show that  $\mathbf{b} \cdot \begin{pmatrix} -12 \\ a \\ -1 \end{pmatrix} = \mathbf{b} \cdot \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix}$ . [2]

(c) Hence, or otherwise, find the value of  $a$ . [2]

8 Two loci,  $C_1$  and  $C_2$ , are defined by

$$C_1 = \left\{ z: |z| = |z - 4d^2 - 36| \right\}$$

$$C_2 = \left\{ z: \arg(z - 12d - 3i) = \frac{1}{4}\pi \right\}$$

where  $d$  is a real number.

(a) Find, in terms of  $d$ , the complex number which is represented on an Argand diagram by the point of intersection of  $C_1$  and  $C_2$ .

[You may assume that  $C_1 \cap C_2 \neq \emptyset$ .] [6]

(b) Explain why the solution found in part (a) is not valid when  $d = 3$ . [2]

**END OF QUESTION PAPER**

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